Circular Part-to-Whole Charts Using the Area Visual Cue

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Abstract

Studies of chart types can reveal unexplored design spaces, like the circular diagrams used in recent studies on pie charts. In this paper, we explore several variations of part-to-whole charts that use area to represent a fraction within a circle. We find one chart that performs very similarly to the pie chart, even though it is visually more complex. Centered shapes turn out to lead to much worse accuracy than any other stimuli, even the same shape when not centered. These first results point to the need for more systematic explorations of the design spaces around existing charts.

1. Introduction

Many studies have examined the retinal variables used in various charts and visualization techniques, such as area, length, etc. The systematic exploration of design spaces around existing charts is quite rare, however. In a studiy on pie charts, we used several such unusual charts to isolate individual visual cues [SK16], one of which unexpectedly performed as well as the pie chart. Might there be other variations that have not been tested and that do as well as, or even better, than the pie chart?

The chart in question, which we called the *area-only* condition, presents the data in a way that should have been harder to read: by drawing a straight line across a circle and coloring one side to represent the fraction purely by area (since the goal of that paper was to discern the visual cue used to read pie charts). A large design space presents itself: curved edges created by circles of different configurations, polygons, concave and convex shapes, etc.

The study presented in this paper examines a subset of these designs. By picking them from a large design space along a few criteria, we selected representatives that sample a variety of ways in which shapes can change as they show different values. Our approach and results are presented and discussed below.

2. Related Work

Pie charts are widely used in the business world, but generally avoided in academic visualization research [Spe05]. They recently received some attention from researchers asking about whether angle was really the visual cue used to read them [KS16, SK16]. Central angle had been established in a paper based on the self-reported visual cue that was never replicated [Eel26]. The newer work suggests that angle is unlikely, and that arc length or area (or some combination) are much more plausible as the retinal variables used.

Not only are pie charts controversial, but studies examining them also come to different conclusions depending on the questions asked and the configurations used. Spence [Spe90], for example, found pie charts to perform as well as bar charts in a study comparing only two values to each other. Pie charts also performed worse than some bar chart configurations in Cleveland and McGill's study [CM84], but better than some others (in particular some of the stacked bar conditions).

However, when comparing parts to the whole, the findings differ. Simkin and Hastie [SH87] found that pie charts were as accurate as bar charts, and better than stacked bars (even though pie charts had performed worst when comparing between slices). Another study came to a similar conclusion when comparing pie, donut, and bar charts to a waffle chart [KZ10]. Hollands and Spence [HS92] also found that pie charts performed better than bar and line charts in their proportion task.

Area perception has been studied in visualization, but there are still open questions. Comparing circles by area as in a bubble chart is known to be difficult [HB10] and sometimes used to misrepresent data [PRS*15]. How differences in shape change this is unclear, however. Kong et al. [KHA10] investigated a number of ways of improving precision when reading treemaps, which encode value by area. They found that squares such as in squarified treemaps [BHvW00] led to more error than rectangles of other aspect ratios. Another surprising finding in that paper was that comparing rectangles of different orientation did not lead to worse error.

3. Circular Part-to-Whole Chart Design Space

Based on the earlier work, we decided to investigate whether variations on the pie chart that all use area as the visual cue (or *retinal variable* [Ber83]) to represent a fraction within a circle, might work as well – or even better – than the classic pie chart. We designed a number of chart variations, largely centered around three ideas:

Pie modifications. These charts would keep the basic pie chart

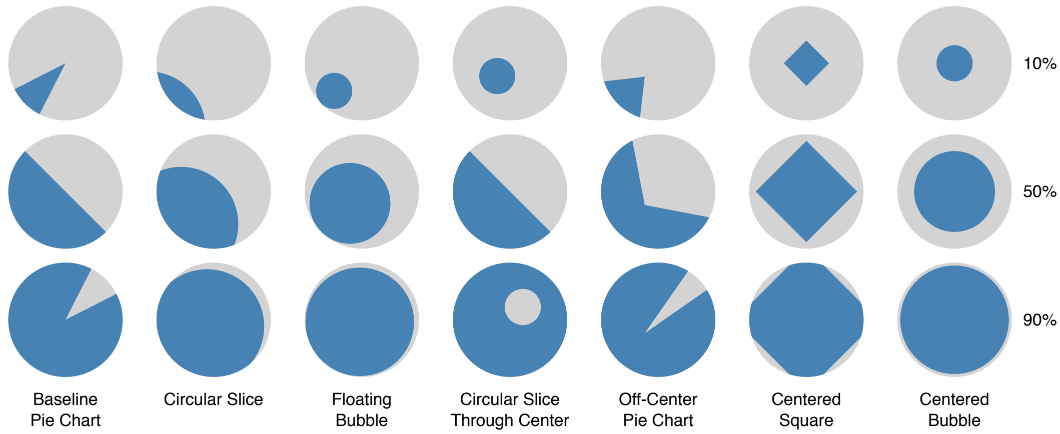

Figure 1: *The seven part-to-whole chart variations tested in the study, ordered left to right by average absolute error (Figure 2).*

Variation	Shape	Center Relation	Value Change	Shape Change
Pie Chart	triangle	attached	grows	no
Circular Slice	circle	none	moves	yes
Floating Bubble	circle	none	grows	no
Circ. Slice Center	circle	attached	grows	yes
Off-Center Pie	triangle	offset	grows	no
Centered Square	square	centered	grows	yes
Centered Bubble	circle	centered	grows	no
Concave Slice	hyperbolic	attached	grows	no
Convex Slice	hyperbolic	attached	grows	no
Circular Segment	segment	none	moves	yes
Rectangular Slice	square	attached	grows	yes
Pentagon Slice	pentagon	centered	grows	yes

Table 1: *A partial classification of the pie-like charts design space. The table lists the shape of the "slice," its relationship with the center of the containing circle, whether the shape moves or grows with the value represented, and whether its shape changes with the value. Variations are listed in the same order as in Figure 1 (the bottom five rows list variations we did not include in the study).*

shape recognizable but make modifications. This includes concave and convex shapes for the slice and off-center pie charts.

Shape overlaps. If the pie chart is considered to be created by intersecting a triangle with the circle, other shapes can similarly represent value by area but with very different shape properties. This includes overlaying circles in different ways (attached to the center, attached to the perimeter, intersecting with the containing circle, etc.), as well as other shapes, in particular regular polygons (attached to the center or not, attached at a corner or along the center of an edge).

Centered shapes. These variations are even further removed from the pie chart metaphor, but they still represent a fraction by area. The most obvious is a centered circle, but any other shape is possible, in particular regular polygons such as triangles, squares, pentagons, etc.

Our initial exploration of shapes led to a partial design space, which we broke down by the shape of the "slice," its relationship to the center of the containing circle, whether it grows or moves, and whether the shape changes as the value changes.

We ultimately picked the following set of representatives from this large design space:

Baseline Pie Chart. A regular pie chart. Edges within the circle are straight.

Circular Slice. This chart represents the value by sliding a second circle of the same size over the base circle. The edge within the circle is curved at a constant radius (the same as the containing circle). In terms of metaphor, this chart is very similar to the way lunar eclipses occur (the Earth's shadow has a constant size that is similar to the Moon's diameter).

Floating Bubble. A smaller circle is attached to the inside perimeter of the circle. The entire smaller circle is always visible. Its border curvature changes with the value and is never the same as the containing circle.

Circular Slice Through Center. A circle representing the fraction is placed so that it touches the center of the containing circle. As it grows, its curvature changes. Once it reaches the border of the larger circle when it shows 25%, it becomes a more irregular shape until it reaches 50%. From there, it turns inside out to contain the smaller fraction, first partially (up to 75%) and then fully surrounding it.

Off-Center Pie Chart. This chart uses a similar shape for its slice as the baseline pie chart, but its center is pushed out by one third of the radius. Angle, area, and arc length are no longer directly related.

Centered Square. The value is encoded in the area of a square in the center of the chart. This leads to the special case where the corners of the square are clipped by the circle when it reaches about 63%. The square's edges are straight, though they are clipped by the circle for larger values.

Centered Bubble. A small circle is drawn on top of the containing circle, and centered on it. The smaller bubble is always visible in its entirety and never clipped, but changes its curvature as it grows.

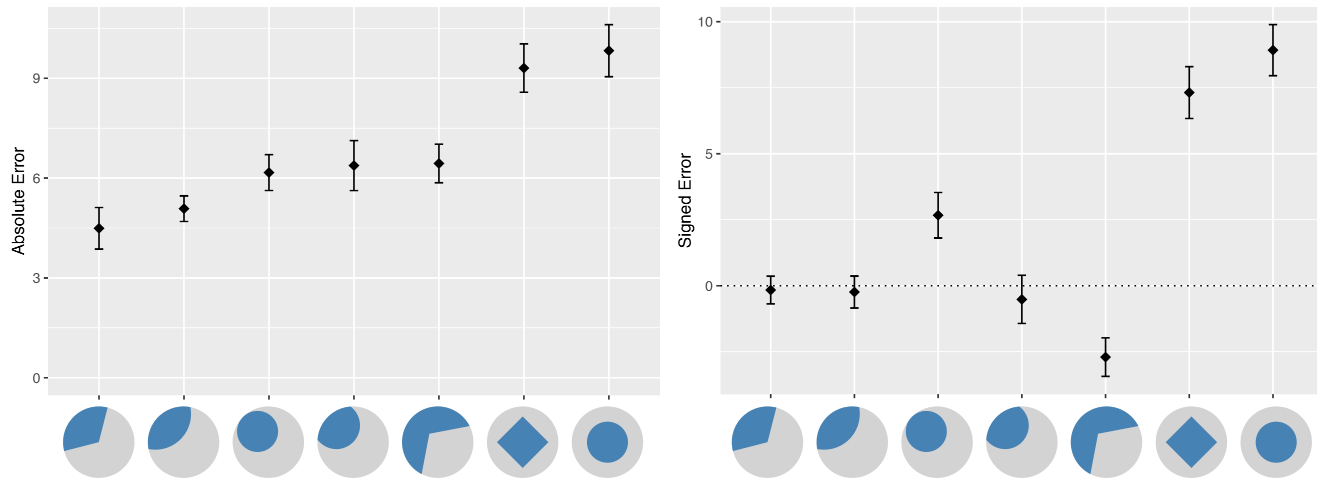

Figure 2: *Absolute and signed error vary significantly by chart type (means and 95% confidence intervals). The only chart with error similar to the pie chart is the circular slice. Circular slice and slice through the center condition also have similar signed error to the baseline.*

Figure 1 shows the variations described above, Table 1 shows the variations and their categorization, including some shapes we considered but were not able to include in this study.

4. Study

Besides all representing a fraction by area, the charts we designed for this study all differ in the shapes that delineate that area and how it changes with the value. We had the following specific hypotheses in mind when we designed them:

- Straight lines perform better than curved ones. The pie chart uses straight lines within the circle, as does the area-only chart.
- Differences in shape help area judgment. Bubble charts are known to perform poorly due to the difficulty with size comparison between objects of the same shape. Perhaps different but regular shapes are easier to compare.
- Centered shapes perform better. The center of the circle is potentially a helpful reference for judging values. The asymmetry caused by a shape that does not always go through the center should make it more difficult to judge its size.

4.1. Materials

When analyzing the data from previous studies, we noticed spikes at values that were multiples of 5. We therefore created four bins of numbers spanning the range from 2 to 97, but left out all multiples of 5, as well as 33 and 66. Each bin ended up with 18 values.

Each participant saw 84 different charts: 7 chart variations $\times$ 4 values $\times$ 3 rotations. Each chart variation was presented twelve times, each with a value drawn randomly from one of the four bins (one from each, three times) and rotated by a random amount. The stimuli were created separately for each participant and their order randomized before presentation to avoid learning and sequence effects. All stimuli represent the value in question as the fraction of the blue area, with the containing circle drawn in a light gray.

Drawing the pie, centered and floating bubble conditions is mathematically straight-forward. For the other conditions, we derived formulas to express the area covered based on a parameter and used a gradient-descent method to determine its value.

All charts were rendered in the browser at a diameter of 600 pixels. While we had no control over how large this appeared on people's displays, we also were in no position to control their distance from the screens. Since the goal was to assess relative judgements, we do not believe that this was an issue, however.

4.2. Procedure

Study participants were given a brief introduction instructing them to judge the percentage shown by the blue (darker) part of the chart. Area was specifically mentioned in order to avoid confusion (especially with the off-center pie chart). During the study, the question displayed did not ask about area, however, but was phrased as *What part of the whole (in percent) does the blue part represent?* (with the words *blue part* also colored blue).

Participants entered their estimate as a number into a text field and advanced to the next step by either clicking a button or hitting the Return key. After 28 and 56 steps, respectively, they were shown a pause screen that encouraged them to take a brief break before continuing. A progress bar was shown across the bottom of the screen to tell them where in the study they were.

4.3. Results

A total of 81 participants (52 male, 29 female) were recruited on Mechanical Turk. Age ranged from the 25–29 bin to 60+, with the majority in the 30–39 range. Education was split almost equally between high-school and bachelor's degrees, with three master's, two other, and one Ph.D. Participants completed the study in an average of 9 minutes and 16 seconds. They were paid $3.00, for an average hourly rate of about $19.47.

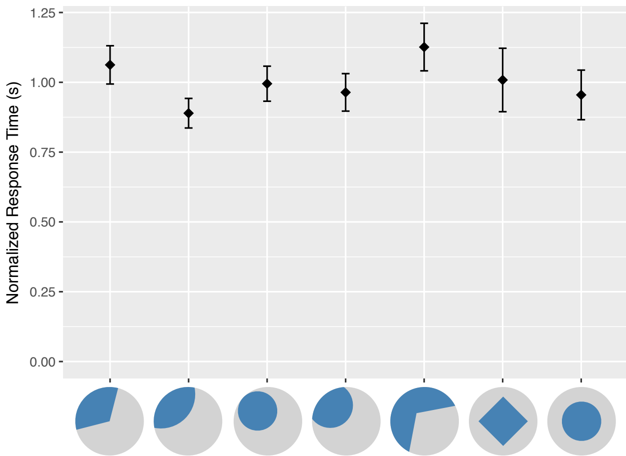

Figure 3: *Response times per question by chart variation, normalized by user. Circular slice charts were faster than pie charts.*

Similar to previous studies [KS16, SK16], we also observed some answers that were apparently for the wrong part of the diagram. However, that number was less than 1% overall, so we decided to not perform any kind of correction. We removed data from two participants whose error rates were much higher than anybody else's, leaving us with data from 79 participants.

We report error in two different ways below: absolute error and signed error. Absolute error is computed as the absolute value of the difference between the represented value and the response from the participant. It serves as a measure of accuracy. Signed error is the difference between represented value and response, which gives and indication of bias (if it is zero or close to zero, there is no systematic over- or underestimation, when it is above or below, there is).

Error by Chart Variation. Absolute error differs significantly between the different charts (ANOVA, $F(6,560) = 41.54$, $p < 0.01$). Figure 2 shows where the differences are: the only chart type with error indistinguishable from the baseline (pie chart) is the circular slice condition, all others clearly perform worse.

Just like absolute error, signed error also differs significantly between the variations (ANOVA, $F(6,553) = 110.5$, $p < 0.01$). The picture is more complicated here, however. The confidence intervals for the circular slice and and the slice through the center overlap the one for the pie chart. The only chart type leading to consistent underestimation is the off-center pie chart, while the floating circle, centered circle, and centered square all lead to overestimates.

Response Time. Response time varies over a large range, both between users and variations. To account for individual differences, we normalized the data first by dividing each duration by the average per user (Figure 3). The differences between the chart variants are statistically significant (ANOVA, $F(6,553) = 3.616, p < 0.01$).

The main difference that is causing this is between the pie chart and the circular slice chart, with the latter being much faster than the pie chart. A paired t-test of the normalized response times between the two chart types confirms this ($t(79) = 3.8935, p < 0.01$). The off-center pie chart is also significantly slower than the circular slice chart ($p < 0.01$), but not compared to the baseline pie chart ($p = 0.32$).

Compared to Earlier Study. We used the published data from the earlier study[†] to compare our results to theirs. Comparing the baseline pie chart conditions, we find no significant difference in absolute (t-test, $t(166.89) = -0.62182, p = 0.53$) or signed error ($t(146.48) = -0.32141, p = 0.75$). The circular-slice chart in this study does not differ significantly from the area-only condition in the earlier one (two-tailed t-test, $t(136.98) = -1.8591, p = 0.065$).

5. Discussion

Almost all the variations we tested in this study did worse than the pie chart. The only exception is the circular slice condition, which does not differ significantly in either absolute or signed error. The area-only condition from the earlier study [SK16] overlaps the pie chart conditions in both studies, so we also consider it indistinguishable from the pie chart.

The two centered shapes showed much worse error than any of the other designs – contrary to our expectation –, and also lead to consistent overestimation (Figure 2). The centered bubble also performs worse than the floating bubble ($t(79) = 10.232, p < 0.01$). This is surprising because both are always fully visible and their curvature changes in the same way, the only difference is that one is centered. The effect is also visible, if less pronounced, when only considering percentages under 25% for the circular slice through the center, where the entire bubble is visible but touches the center; it also has lower absolute error than the centered circle in this case ($t(79) = 2.1914, p = 0.031$). It appears that centering has a negative effect on area comparison.

The amount of change in shape also appears to have little effect on error. The floating bubble changes its curvature, and the circular slice through the center and the off-center pie chart change their shapes considerably over the range of values. It is hard to say whether that has an impact, since those charts do have somewhat worse performance, but they still perform better than the two centered shapes. Straight lines by themselves do not appear to make reading the charts more precise (as we would have expected).

One limitation of this study is that we only used two-slice charts. We do not believe that more slices would change our major findings. Multiple slices were tested for the circular slice and other conditions in a related paper [Kos19].

6. Conclusions

How did the pie chart emerge as a popular chart type? It lives within a huge design space that has not been explored so far. Our study shows that at least out of the variations we tested, all but one performed worse. Given our findings on centering and the circular slice chart (as well as the *area-only* condition from the earlier study), we believe that more research is needed to understand the underlying perceptual mechanisms of the pie chart and its relatives.

† https://github.com/dwskau/arcs-angles-area

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