Shapes on a plane: Evaluating the impact of projection distortion on spatial binning

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Abstract. One method for working with large, dense sets of spatial point data is to aggregate the measure of the data into polygonal containers, such as political boundaries, or into regular spatial bins such as triangles, squares, or hexagons. When mapping these aggregations, the map projection must inevitably distort relationships. This distortion can impact the reader's ability to compare count and density measures across the map. Spatial binning, particularly via hexagons, is becoming a popular technique for displaying aggregate measures of point data sets. Increasingly, we see questionable use of the technique without attendant discussion of its hazards. In this work, we discuss when and why spatial binning works and how mapmakers can better understand the limitations caused by distortion from projecting to the plane. We introduce equations for evaluating distortion's impact on one common projection (Web Mercator) and discuss how the methods used generalize to other projections. While we focus on hexagonal binning, these same considerations affect spatial bins of any shape, and more generally, any analysis of geographic data performed in planar space.

Keywords. Spatial binning, map projections, visual analysis

1.0 Introduction

Maps present clear value for understanding spatial relationships and patterns. On the one hand, there are base maps that focus on topography, hydrology, and natural resources. On the other hand, there are thematic maps that convey relationships between topical phenomena. In both cases, the communication relies on interpretation of spatial relationships, and so the success of the map depends on the accuracy of placement and symbolization within that space. It is both a benefit and a hazard that maps reduce the complexity of real world distributions into point, line, polygon, and raster representations. While this reduction simplifies patterns for interpretation and communication, the mapmaker must be careful to consider how simplification affects accuracy of interpretation. Examples of simplifications include mathematical or statistical manipulations of the raw data, regrouping (classifying), and any analyses involving measurement or use of area, distance, or direction between data elements.

The work reported here highlights issues in simplification of large N point datasets into spatial bins. In this work we focus on not only how spatial binning can improve our ability to analyze spatial patterns for large and dense point datasets, but also how it may hinder a reader's ability to interpret meaning in the transformed data. We emphasize the importance of geometrical distortion of space as a function of map projection transformations, as this is the problem with the least scholarly discussion in binning, has the greatest potential impact on visual and spatial analysis, and has been largely ignored in discussions of vector spatial binning for point-based geographic data. While we are focusing on issues related to aggregation of point data, it would be remiss to ignore mentioning to the reader the small, but interesting overlap with the corpus of literature on pixel distortion for raster re-projection (e.g., Steinwand, Hutchinson, and Snyder 1995; Mulcahy 2000; Usery et al. 2003, White 2006). Though the aggregation problem in vector spatial binning is quite different, it is interesting to consider the challenges of resampling that result from raster projection transformations.

While we cover general implications of map projection on this type of data manipulation, we will focus on one particular projection, the Web Mercator. For better or worse, we are in an era of general purpose thematic mapping using the Web Mercator. While not the only projection used in modern mapping, it is the projection of most Web mapping systems (e.g., Google Maps, Bing Maps, ArcGIS Online, Yahoo! Maps, MapQuest, etc.) that allow users to add their own thematic content.

The goal of this manuscript is to explain when and why spatial binning is appropriate for geographic data, and, importantly, what trade-offs mapmakers need to understand in order to aggregate in bins and to communicate the results appropriately.

We approach the discussion with the following considerations about projections:

- For projections that are equal area, it is possible to preserve area in spatial bins. However, these bins will *not* be regular (same shape) on the sphere.
- For projections that are not equal-area, regular spatial bins in projected space will not express regular area measure. Therefore, the mapmaker must decide whether the bins will be used to present comparable measures for densities (quantity per unit area *on the sphere*) in the resulting bins, or if they will be used to present quantity (Figure 1).
    - If the bins measure density, then the reader may be hindered in estimating quantity because the spherical areas of the bins will vary across the map while the size and shape of the bins remain the same. If two bins show the same density measure, for example, readers may be led to also interpret counts as being the same when they may in fact differ drastically.
    - If the bins measure quantities then the ability of a reader to compare densities may be hindered for the same reasons; the planar area of the bins differs from the spherical area, whereas a naïve translation of density would assume that the regular bins represent equal areas. Since this is false, except for equal-area projections, the presumed density would be incorrect.
- For any projection it is possible to create *irregular* spatial bins that reflect true area on the sphere. The irregular bins can be designed in such a way as to preserve both density (quantity per unit area on the sphere) and quantities. However, the extent of the bins will vary when projected onto a planar map, negating what we feel is the visual benefit of the regular bins (**Error! Reference source not found.**). Even with this practice, how readers might interpret the data is an open question. Consider an area in the high latitudes that has been greatly expanded on the Web Mercator in order to show the same amount of area on the globe as a region near the equator. If the hue used to represent the quantity is the same in both regions, then the represented density would also be the same. And yet the reader would see a vastly expanded area full of color and might thereby infer greater quantity by thinking the larger bin meant larger ground area.
- The map reader may not identify or compensate for distortion introduced by the map projection, leading the reader to assume that equal count implies equal density even when bins of the same size on the plane represent different sizes on the globe.

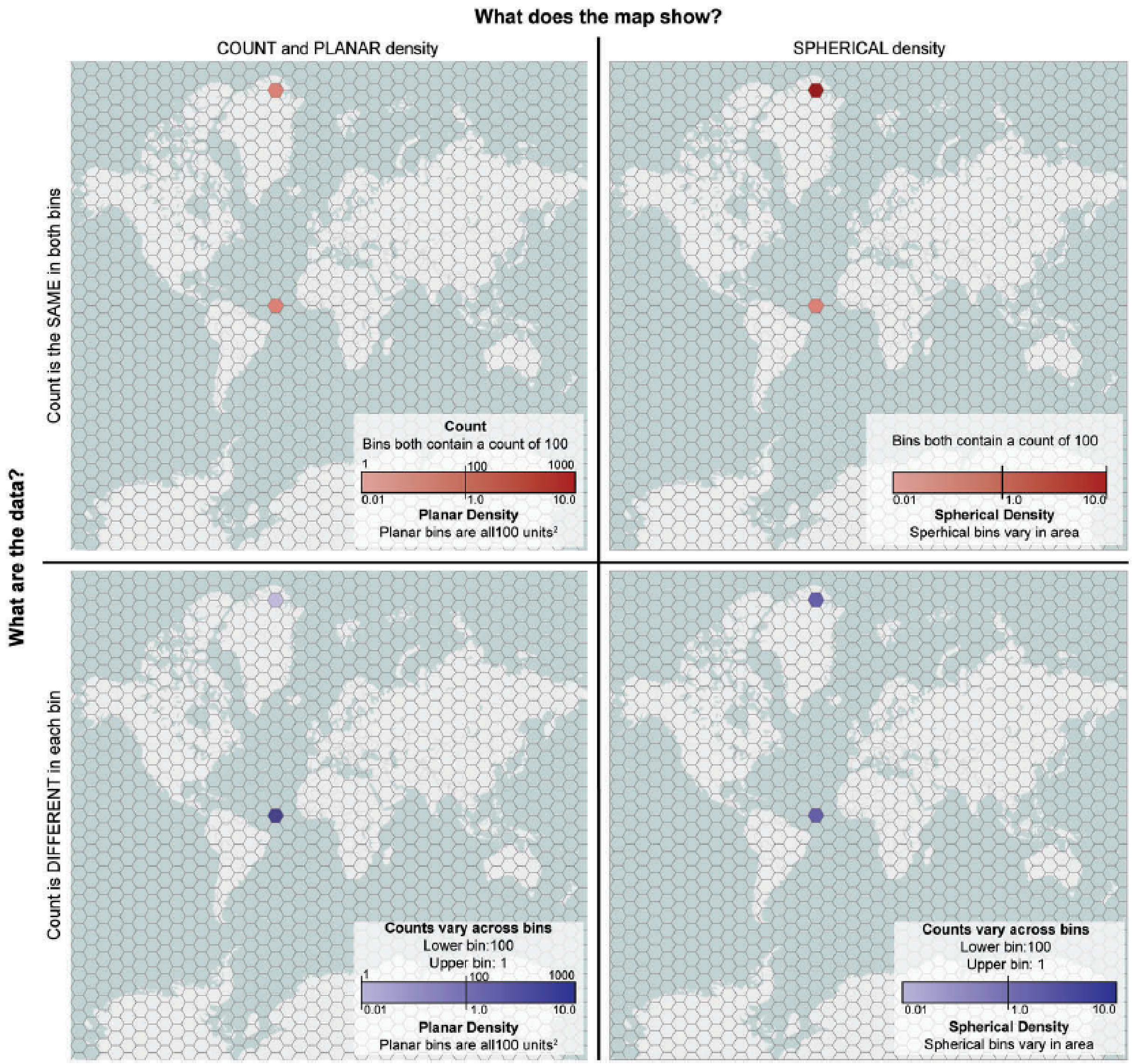

Figure 1. Regular spatial bins on a nonequal area projection (Web Mercator in this example) present challenges for users to appropriately estimate both quantity and density in the same visualization. If the reader assumes that the spatial bins represent the same area, the assumed density values will also be equivalent for both locations (left column). However, if spherical area is (more appropriately) used to calculate density, as seen in the right column, it is apparent that the density is different.

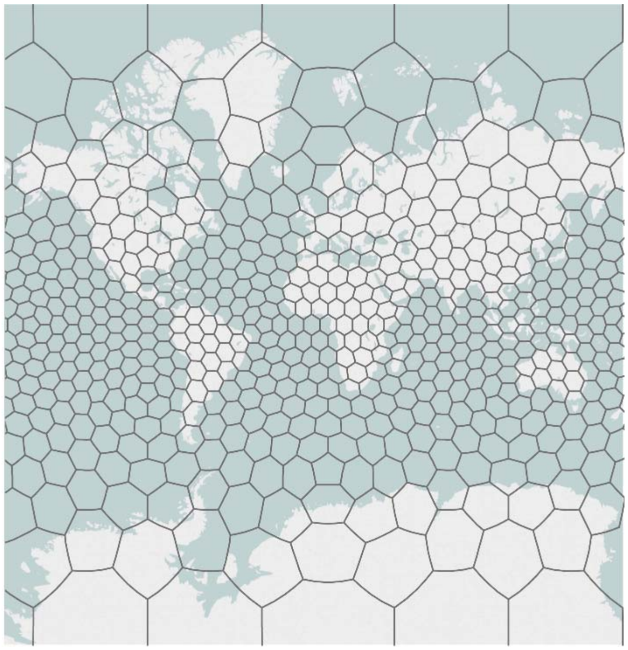

Figure 2. Icosahedral Snyder Equal Area aperture 3 Hexagon bins at resolution level 3 shown in the Web Mercator projection.

2.0 Spatial binning

Spatial binning is a method of aggregating individual point locations into polygonal regions. These polygonal regions may be irregular, and are often political boundaries (e.g., census tracts or countries) or a tessellation of regular polygons. Spatial binning using irregular polygons is one of the most common forms of mapping, in which we create choropleth maps by counting discrete entities or deriving a new measure for each polygon from the raw data. Examples are population density or median household income for a census tract as derived from individual income combined with average household size.

Binning using regular polygons appears in the literature from the early 1980s at latest, with Chambers et al.'s (1983) sunflower plots and Carr et al.'s (1987) discussion of hexagonally binned plots for large-N datasets in scatterplots. Carr, Olsen, and White (1992) were possibly the first to discuss explicit application of hexagonal binning for spatial data aggregation and sampling. Since this time, there has been little scholarly discussion of theories of hexagonal binning for spatial and visual analysis. However, numerous examples of their use appear in recent academic literature and popular media. In the academic literature, discussion has focused on using rectangular and hexagonal bins to aggregate data for analysis or display (Schipper et al. 2008; Hoffman et al. 2010) as well as for constructing sampling frameworks (Birch, Oom, and Beecham 2007; Elsner, Hodges, and Jagger 2012). Popular media examples largely revolve around the use of hexagonal grids for binning (for instance Goldsberry's [2012] basketball visualizations and Field's [2015] map of deaths in the Grand Canyon) or as "tile grid" maps (see DeBelius [2015] for discussion), as well as how-to guides (Field 2012; Graser 2012) justifying and demonstrating the method.

In the remainder of this section, we discuss the nature of geographic phenomena and data appropriate for spatial binning, as well as the attending benefits and hazards that may hinder communication of results.

2.1 Why use regular spatial bins?

Use of regular spatial bins has three primary goals. The first is to simplify a dataset, which may help improve rendering speed. With sufficiently large datasets, it may be quicker to store and draw a small number of polygons than it is to render the individual points. Because points may not be individually distinguishable anyway, bins provide a second benefit in aiding visual communication. Done properly, binning can permit the reader to make reasonable count or density estimates that otherwise would be impossible because point symbols have coalesced or overlapped (e.g., **Error! Reference source not found.**). Binning is also useful for simplifying patterns in which mark overlap may not be excessive by showing a smoothed surface of aggregated values rather than requiring the reader to mentally aggregate. The third goal is to provide a regular, gridded framework for additional analysis or comparison between datasets (e.g., as seen in Data Team 2015). Spatial bins can also be used as a framework for sampling (e.g., White, Kimerling, and Overton 1992); the same concerns about accuracy in spatial binning apply for this use case.

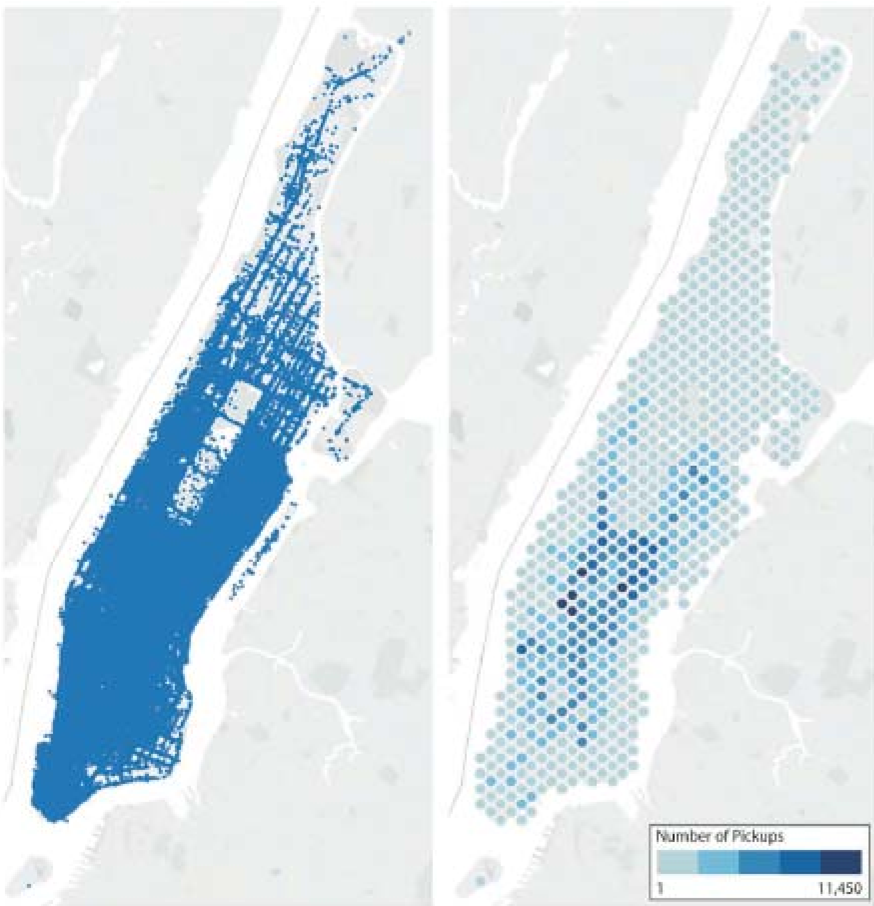

Figure 3. Taxi cab pickup locations in Manhattan as raw point locations (left) and as counts after being binned into a hexagonal grid (data from Andres Monroy - http://www.andresmh.com/nyctaxitrips/).

Regular spatial bins displayed on a map are effective for visual analysis because they allow us to hold shape and spacing in the symbols constant, eliminating irrelevant variation in the symbols. This brings the reader's attention to the visual encodings, such as size or color, which indicate variation in the relevant measure (see **Error! Reference source not found.**). Though any of three different shapes (triangle, rectangle, hexagon) can be tiled for regular binning (tessellation), hexagons appear to be a favorite since they are considered more attractive (Carr et al. 1997; Carr 1990), show lower density estimate bias than squares or triangles (Scott 1988), are less likely to line up with regularly spaced and linearly shaped cultural features (Carr, Olsen, and White 1992), and are less likely to lead readers to "see regularity where there is none" (Wilkinson 2005, 142).

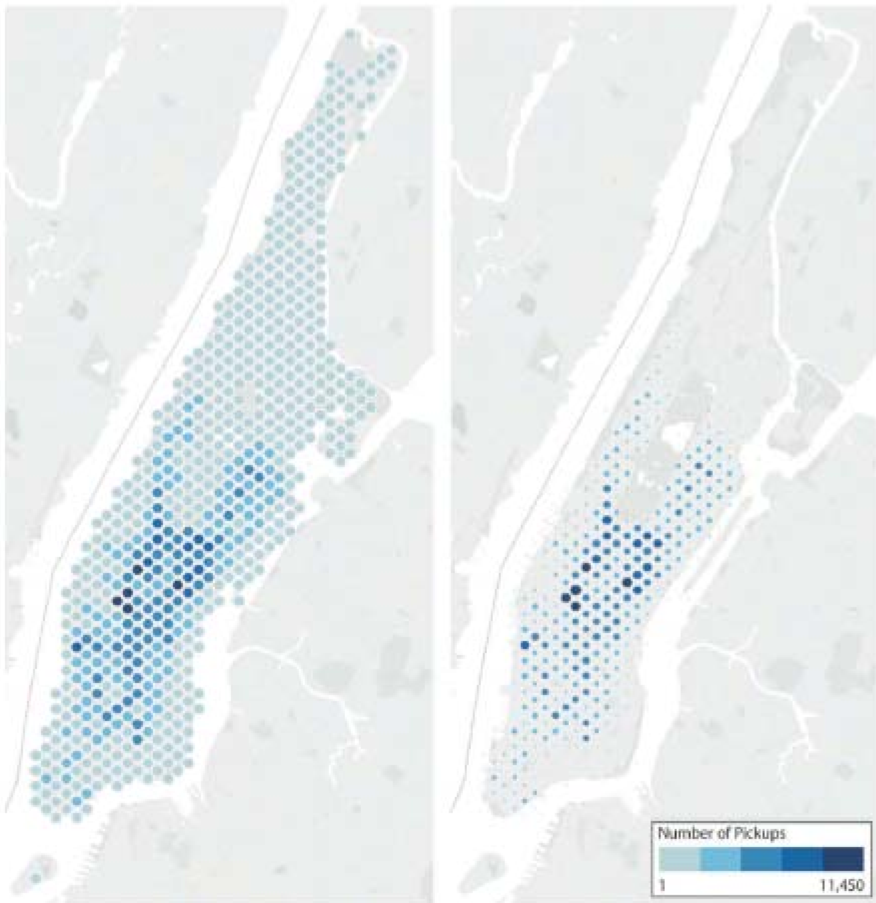

Figure 4. Hexagonal bins color encoding only (left) versus size and color encoding (right).

2.2 For what type of geographic phenomena and data is spatial binning appropriate?

Spatial binning is appropriate for aggregation of point-based measures where it is important to maintain the proximity of the measure to the location of that measure. When aggregating into regular polygons, the fact that they are all the same size implies that they function as a density measure, however, that may be mere appearance, not necessarily a representation of actual density. We will come back to this important point in the next section. This type of aggregation is most appropriate for geographic phenomena that vary reasonably smoothly over space and that fit somewhere between discrete and the middle of the discrete-continuous continuum, as defined in MacEachren and DiBiase's (1991) model of geographic phenomena.

As discrete data, points can then be aggregated for visualization as counts (or other calculations such as sum of an attribute) within each of the spatial bins. This is the same process of aggregation as any point data into polygonal geographies (e.g., sum of population within a census tract), though since the polygons are the same size they become a density cue rather than purely a count.

Though there are examples of spatial binning used to visualize aggregation of non-point data (e.g., Schipper et al. [2008] for polygonal habitat regions; Elsner, Hodges, and Jagger [2012] for hurricane paths), spatial binning may not be the most appropriate method for clearly communicating these patterns of spatial density. For such data, binning into regular polygons is more complicated because the process requires preprocessing to identify what to add, and to quantify how much of each line or polygon to add, into each bin. Decisions made to quantize the data, such as ignoring slivers to minimize over-count, might obfuscate the meaning of the measure for the reader (though we have not tested this). There is also the potential for a reader to presume an even distribution of the phenomenon when, in fact, it has high variability. For instance, when mapping species habitats, do we assume the probability of finding an example of the species as constant across the entire polygon?

2.3 Comparison of spatial binning to other related mapping methods

Spatial binning is not the only cartographic method that can be used to analyze and visualize point density. Several other methods are in use; however they have subtle and not-so-subtle differences. While other methods also help simplify the display, they are not appropriate for all geographic phenomena. In this section we evaluate how well these other methods meet the goals we defined for spatial binning.

Choropleth. A popular choice, choropleth maps constrain counts of points to irregular polygons that were likely defined independently of the phenomenon. They are best used for phenomena distributed evenly and continuously across an enumeration unit, and that change abruptly along the borders of the enumeration unit. When used for aggregated count data, they are most appropriate when enumeration units are similar in size or when the count has been normalized to account for variation in other attributes of the enumeration unit (Slocum 2009). Spatial binning with regular polygons and the irregular polygons of choropleth maps are similar; however, regular polygons have a few advantages. For one, regular polygons are not tied to political boundaries. That allows us to show smoother patterns of change over space, which is important for phenomena that are not naturally constrained by anthropogenic boundaries. Also, the size of regular polygon bins can be adjusted to provide greater control over the representation of the data points' distribution, which mitigates masking uneven distributions of the mapped phenomenon. Finally, regular polygons provide a uniform grid of enumeration units for either visual analysis or spatial analysis – though possibly not both. We discuss this last matter in more detail later in the manuscript.

Dot maps. While choropleth maps constrain counts to polygons, dot maps are not constrained in this way; they can use arbitrary regions of any complexity to aggregate and show distribution. In constructing the map, the mapmaker must decide how to distribute the smoothly changing phenomenon realistically, choosing enumeration units for that purpose and choosing a fixed quantity of the phenomenon that each dot will represent. This technique differs from hexagonal binning in that the enumeration units are not evident to the reader and are not uniform. Therefore the reader cannot infer real locations for the phenomenon, with the distance of a particular instance of the phenomenon to its dot being arbitrary and potentially large.

Heat maps. With heat maps, the input point data are the same as for hexagonal bins, but the output goal is to generate a continuous surface of density by filling in gaps between points with interpolated values and removing the attachment of points to exact locations. Geographic heat maps, or kernel density estimation maps—as opposed to statistical heat maps as discussed by Wilkinson and Friendly (2009) —use interpolation to transform discrete point data into a continuous density surface. Each location is assigned a measure representing the count of points found within a fixed radius. This method provides a smooth representation of density for a phenomenon across the entire surface, but removes the original data points from the visualization. For large datasets, creation of heat maps becomes computationally intensive (Shook et al. 2012).

Isopleth maps. Another option is to create a kernel density surface or to interpolate a surface from a measure at point locations and then translate this into a set of contour lines that connect locations of equal measures. The contour lines are isopleths (lines of equal value), so the map itself is known as an isopleth map, although the terms isopleth and contour are often used interchangeably (Robinson, Sale, and Morrison 1978). While this also provides a smoothed representation for the phenomenon, similar to heat maps and spatial binning, the resulting map, once again, is more computationally intensive. More importantly, this generates *new values* between known points as a consequence of interpolation, implying a truly smooth, continuous change of attribute, as opposed to simply aggregating values based on their containment within a bin.

The literature describes other mapping methods, such as proportional symbols using true or conceptual locations, dasymetric, or chorodot (MacEachren and DiBiase 1991). However, as they pertain to spatial bins, we see only nuances in these other methods beyond the contrasts already discussed above.

2.4 Creating spatial bins

Spatial binning seems like it should be simple: create a grid of triangular, rectangular, or hexagonal bins on the map and then count the points contained by these bins. It is a mere point-in-polygon exercise. However, it turns out that map projections cause problems, particularly at small cartographic scale. Essentially, we have to decide whether the bins are regular polygons on the sphere (or ellipsoid or geoid) or on the plane. The decision here substantially impacts the resulting analyses.

Unfortunately, as spatial binning (particularly hexagonal binning) has become more popular in recent years, the focused discussion of these problems from the 1990s (e.g., White, Kimerling, and Overton 1992; White et al. 1998, and Kimerling et al. 1999) has dissipated. The current discussion seems to be focused largely in the popular media on how hexagonal binning can provide accurate visual representation for analysis of geographic data density (e.g., Smith 2012; Briney 2014). Guidance on how to create and use spatial bins tied to GIS software (Field 2012; Graser 2012), statistics packages (Carr 2015; SPSS; Wicklin 2014), and Web mapping libraries such as d3 (Bostock, Ogievetsky, and Heer 2011) and cartoDB (cartoDB 2015) provide little to no commentary on what the spatial bins *mean* to a reader or how they could be misleading. Additionally, the popular media documentation has typically just described hexagonal binning as a method for simply and effectively representing complex datasets so that they are easier for users to understand (e.g., Smith 2012; Briney 2014) without concern for the visual analysis challenges introduced by non-equal area bins. In our searches, we have found few sources in the popular media that even mention projection, although Nelson (2013) nicely reminds blog readers to use an equal area projection because "this is pretty much the whole point."

Spatial binning has been subjected to little criticism in the academic literature beyond the previously mentioned work by White and others, and a few efforts to minimize the projection effects for planar binning using hexagon-based discrete global grid systems. Most notable in influencing the type of bins used in scholarly literature may be the work by Sahr, White, and Kimerling (2003) to construct the Icosahedral Snyder Equal Area-based hexagonal (ISEA3H) geodesic discrete global grid (for application examples, see Strassburg et al. [2010]; Schipper et al. [2008]; Hoffman et al. [2010]). We discuss this and a few other spherical binning methods in the next section.

2.5 Problems with bins in spatial and visual analysis

To draw attention to the problems caused by map projection that may undermine spatial bins for aggregating and visualizing data, we consider the case of hexagonal binning for display in Web maps using the Web Mercator coordinate system. This is not an arbitrary use case; in fact, hexagonal binning on Web maps is a practice we see frequently and is one that is particularly tricky to do well.

As we mentioned earlier, regular spatial bins on a map are effective for visual analysis because they hold shape and spacing in the symbols constant. This works best when the bins are, in fact, regular on the plane in large-scale mapping cases (**Error! Reference source not found.**). In order to maintain this regularity of bin shape and size on the plane, we must compromise the shape or size of the bin, or both, *on the sphere*. Representation of surface area is particularly important to consider because if the area measure on the sphere varies while the bin's planar area holds constant, then a reader's ability to assess density will be compromised. To demonstrate this, we show the impact of two different – but common – methods for creating hexagonal bins: regular planar bins defined with projected map coordinates *x,y* (meters in Web Mercator), and bins based on "spherical units" as defined on a plane (e.g., degrees of latitude and longitude in the equirectangular plate carrée projection). The first of these methods creates a regular lattice of hexagonal bins on the plane such that they appear to be the same size and shape, and yet the territories they represent on a sphere differ in area. The second method is a naïve attempt at generating regular bins on a "sphere." This method treats spherical coordinates as planar, a practice that approximates regularity for neither spherical nor planar bins. Additionally, we briefly discuss a third method to create bins on the *sphere* and then translate these to planar coordinates. Basic properties of these methods are summarized in Table 1.

Table 1. General properties for global-scale regular spatial bins defined with different methods.

	Mapped space (plane)		Spherical space	
	Same size	Same shape	Same size	Same shape
For regular bins defined on the…				
Plane – with equal side length as defined by				
Map units, on an equal area projection	✓	✓	✓	-
Map units, on a non-equal area projection	✓	✓	-	-
Degrees of latitude and longitude*	-	-	-	-
Sphere – projected to the plane	?	?	?	?

✓ Feasible to create bins with this property, but preservation is *not* guaranteed. Magnitude of deviation depends on projection, location, and spatial extent of bins (e.g., Figure 1 for a non-equal area projection example)

\- Not feasible to create regular bins with this property at global-scale. Regular bins on a non-equal area projection will not be regular shape or size across the sphere.

? Feasible to maintain some of these properties, but it depends on the method of subdivision of the sphere, projection to the plane, bin shape, and area covered (e.g., see White et al., 1998 and Carr et al., 1997). Regardless of subdivision method, translation to mapped space will always introduce distortion of area and or angular measurements.

* However, if created and used on a plate carrée projection the bins will be same size and shape on the plane; any other projection will present differing sizes and shape of bin (see **Error! Reference source not found.**)

Defining planar bins with projected units. For planar bins, coordinates on the *x*- and *y*-axes are treated as true distances. From that, bins of equal side length are generated. This gives a regular lattice of equal shape and size hexagons. On the sphere, however, these bins can represent very different areas depending on the projection, the spatial extent of the data (with larger geographic areas being more likely to present substantial deviation in area), and the map scale. In Web Mercator coordinates, this means that the bins farther from the equator cover smaller and smaller ground areas even while all having the same projected area. This occurs because the scale factor in the Web Mercator projection increases with latitude, wherein scale factor is proportional to secant of latitude (Snyder 1987). This leads to an apparent reduction in the density of data at higher latitudes when the points being binned are distributed evenly (e.g., **Error! Reference source not found.**).

Figure 5. A spatially uniform distribution of one million random data points as binned into even-sized regions in Web Mercator planar coordinates, shown with color (left) and color/size encoding (right) to emphasize the relative change in density. The ground area covered in the high absolute latitudes is much smaller than the area covered by the same size bin near the equator, leading to an appearance of lower density nearer the poles.

Defining planar bins using degrees of latitude and longitude. Another dubious technique we have encountered is to define bins using degree "units" to define hexagon side length, with the length of a degree of longitude and latitude considered equivalent. The problem here is that the length of one degree of longitude is not constant across latitudes; it varies from ~112 km at the equator to 0 km at the poles. When the bins are then projected into Web Mercator coordinates, significant distortion to bin shape and size may be introduced, with severity of the distortion dependent on latitude. This method of defining hexagonal bins introduces even more significant errors than the regular planar grid discussed above because the projected bin size becomes larger and larger as the absolute latitude increases, while the ground area covered by the bin becomes smaller and smaller (**Error! Reference source not found.**). The bins are regular in neither spherical space nor mapped space.

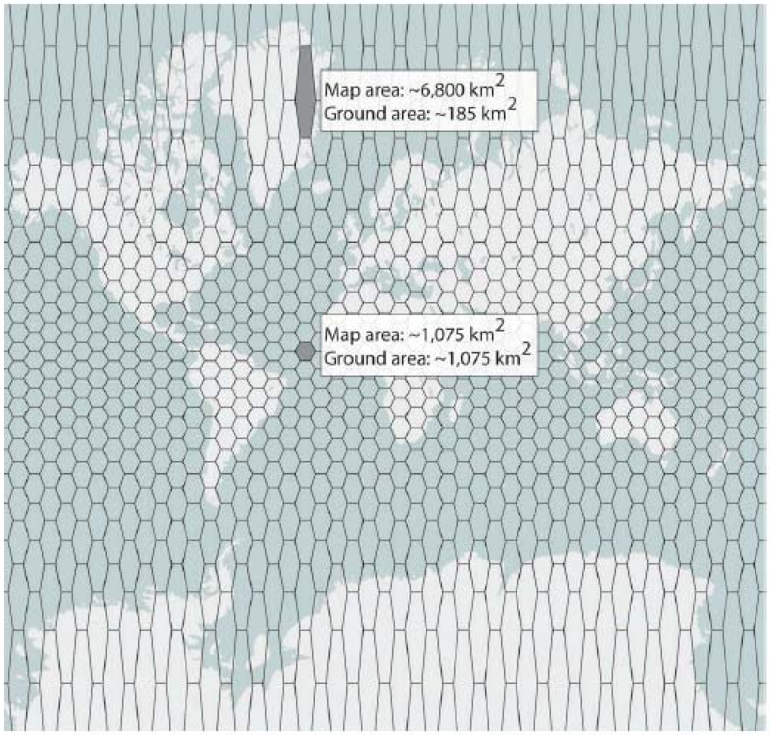

Figure 6. Hexagonal bins defined as 10° side on a plate carrée projection, and projected onto a Web Mercator base map. Measurements listed provide relative map area and ground area.

Defining spherical bins and projecting to plane. A third method of generating regular spatial bins is to define them on the sphere and then project to the plane. There are many ways to partition the surface of the globe to generate grid systems. Specifically with respect to tessellations used for sampling frameworks (or spatial binning) that consist of equal area and shape units, White et al. (1998) explore troubles that arise with methods of recursively partitioning the globe. They considered three methods of defining spatial bins: 1) planar partitioning that were then projected back to the sphere (using gnomonic, Fuller Dymaxion, and Snyder equal area polyhedral projections), 2) spherical subdivision of an icosahedron inscribed in the sphere, and 3) polyhedral partitioning with the mapping between plane and sphere changing at each level of recursion. Of all of these methods, only the Snyder equal area projection maintained equal area across all subdivisions, though shapes were not preserved. The Fuller projection and direct spherical subdivision provided the best balance in distortion between area and shape. Sahr, White, and Kimerling (2003) continued work with the Snyder equal area projection-based partitioning and constructed a geodesic discrete global grid system called the Icosahedral Snyder Equal Area aperture 3 Hexagon (ISEA3H) that can be used for binning. However, while these bins are equal area on the sphere, when projected onto a plane they will be warped in the transformation and likely will not still present a pattern of regular shape and size bin (**Error! Reference source not found.**). While the ISEA3H has been directly discussed in the literature, any equal-area projection yields bins of equal size on both sphere and plane; common examples are Eckert IV (terrestrial) and Mollweide (astrophysics, oceanography). Shape, on the other hand, varies across the map.

Interpreting meaning from the symbolized bins. In addition to the challenges of creating bins appropriate for the needs of the map, the mapmaker must also consider how symbolization affects the interpretability of the measure assigned to each bin. In a way this may present a "benefit" from the errors inherent in spatial binning: Any inaccuracy in raw quantity or density measure may be hidden, or dampened, within a range of values in a classed dataset or behind the just noticeable difference of shade in an unclassed dataset. This is not to recommend classification of data and reporting of data range as a solution for bad spatial binning, only to observe that other factors also impact a map reader's ability to interpret the patterns presented in the spatial bins. Above all, we believe the mapmaker should focus on the accuracy of the raw data while keeping in mind how classification and symbolization affect interpretation.

3.0 Calculating 'safe zones' to bin in projected space

To use spatial bins in planar, projected space, the mapmaker needs to identify the 'safe zone' for which the error from binning does not exceed some defined tolerance. As Web Mercator is a common projection for online maps, we show how to calculate the safe zone for this projection and then discuss how one could determine the safe zone in other projections.

The Web Mercator has uniform scale along parallels. To create bins without violating some acceptable limit of error, the map designer should designate a standard parallel (a latitude) of interest and then compute the north-south zone around it that does not exceed the acceptable error. The idea is that the standard parallel is the reference scale. It has no "distortion." Viewing any particular latitude this way is permissible on a conformal, or essentially conformal, map like the Web Mercator because sizes are purely relative and there is no absolute distortion of shape as there is on non-conformal projections. There is only inflation or deflation of scale (which we refer to generically as "flation") from whatever is designated as the standard. Considered this way, the nominal scale of the map is defined by the standard parallel's scale rather than by the equator's scale.

The safe zone around the standard parallel is characterized by a maximum tolerable flation that is presumably close to 1.0, where 1.0 represents the standard parallel's scale factor. Flation is greater than 1.0 in the direction of increasing scale on the map, and less than 1.0 in the direction of decreasing scale. To allow 5% error away from the standard parallel, for example, *flation* would be 1.05 in the direction of the nearest pole and 1/1.05 = 0.9525 in the direction of the equator. We select 5% as the example error value as it is in line with the 0.06 Weber Fraction for just noticeable difference in visual area (Baird and Noma, 1978), however, the reader has freedom to select an error value in line with the quality needs for their data.

Given an allowable flation k and the standard parallel φ_1, then the maximum difference (δ) in latitude that does not exceed the permissible flation k is found as:

$$\sin\delta = k^{-1}\cos\varphi_1\,[\sqrt{(k^2 - \cos^2\varphi_1)} - \sin\varphi_1]$$

$$= (\sqrt{[k^2\sec^2\varphi_1 - 1]} - \tan\varphi_1) \,/\, (k\sec^2\varphi_1) \qquad (1)$$

To find the upper and lower boundaries of the zone, k would take both the upper and lower desired flation values. These formulae are derived from the trigonometric identities that express the secant of the sum of two angles (φ_1 and δ), setting that equal to k, and solving for δ. The secant of latitude represents the flation on a Mercator map considering the equator as 1.0.

To find the relative flation k at $\delta°$ from φ_1:

$k = \cos(\varphi_1)/\cos(\varphi_1+\delta)$ (2)

This follows directly from the flation value calculation on the Mercator. Note that this expression is only applicable on one side of the equator at a time. When spanning the equator, the equator itself is φ_1, and whichever parallel φ_{max} is furthest from the equator is δ, reducing the formula to:

$k = \sec(\varphi_{max})$ (3)

Table 2 shows a series of safe zones on Web Mercator given an upper k of 1.05 and a lower k of 1/1.05 = 0.9525, with standard parallel for each. This table is intended as an illustration, not as a prescription, since a responsibly exploited zone depends on the area of interest, not this arbitrary subdivision into bands of latitude. As expected, the usable bands narrow dramatically with latitude, starting at 35.5° at the equator but shrinking to a mere 2.3° at the Arctic Circle. Also as expected, the half of the band closer to the equator from a given φ_1 is wider than the half of the band away from the equator.

Table 2. Example series of 'safe zones' on Web Mercator with an assumed flation tolerance of 5%.

low φ	–17.75°	17.75°	30.25°	38.42°	44.71°	49.86°	54.22°	57.97°	61.25°	64.13°	66.69°
φ_1	0°	24.90°	34.64°	41.74°	47.40°	52.13°	56.16°	59.66°	62.73°	65.45°	67.86°
high φ	17.75°	30.25°	38.42°	44.71°	49.86°	54.22°	57.97°	61.25°	64.13°	66.69°	68.96°

While we demonstrate for the Web Mercator, similar concepts apply to any projection, albeit complicated by whatever vagaries of distortion the projection might have beyond simple latitudinal zones. For equal-area projections, the question is not "Where do things get too big or small," but, "Where do my polygons start looking too skewed from angular deformation?" For compromise projections, both considerations come into play, where one would impose limits for both flation and angular deformation.

4.0 Conclusions and further work

As pointed out by O'Sullivan (2015) in a recent book review, "…Addressing aggregation effects in spatial data remains a relatively unexplored area." In this work, we have addressed one component of these effects, that of spatial binning, which presents hazards for creating bins that are both accurate and easily understood by the map reader. We have discussed many aspects of spatial binning including how to assess whether binning is even a good choice, as well as observations about problems caused by map projection distortion, and how to mitigate these problems. Part of assessing whether or not regular spatial bins are a good way to visualize a dataset depends on what projection is used and how it is used, so we have introduced the notion of a "safe zone" in which the distortion from the map projection is minimal enough to avoid compromising information integrity. Though we have focused on examining, and helping mapmakers understand, the problems map projections bring into spatial binning, there remain many related topics ripe for research that should be considered more deeply when using spatial bins to aggregate data.

We have not discussed some relevant, important geographic and cartographic concerns that are already well known and well documented in the literature. We mention them only briefly here to remind the mapmaker that these concerns should inform analysis and design decisions, and to introduce interesting future research questions.

For one, any map using spatial binning is affected by the modifiable areal unit problem (MAUP) (Openshaw and Taylor 1979). The MAUP is a result of aggregating point data into polygonal bins, whose boundaries artificially partition the data and thereby affect the calculated summaries. Implications of the MAUP have been widely discussed in the literature (e.g., Fotheringham and Wong 1991; Grasland and Madelin 2006; and many others), so we do not go into detail here, but simply remind the reader that analytical values and visual patterns resulting from the aggregation vary depending on the size, shape, and placement of the bins. While the problem is unavoidable, it is not ignorable. The effect is particularly pertinent when the goal is to compare patterns across multiple maps employing the binning technique: If the bins used in both maps are not the exact same size and placement, the patterns will not be directly comparable. We have found that not all tools for creating bins allow for easy and explicit definition of the bin size and origin location.

The MAUP in binned data suggests to us that raw counts may not be the best way of partitioning data between bins. We see potential for techniques analogous to anti-aliasing from the signal processing domain. Aliasing happens when a continuous signal is quantized into regular measurements of intensity, such as digital sampling of an audio signal. The loss of detailed structure within each sample, combined with the edge of the bin, means that the original signal cannot be perfectly reconstructed, that different sampling frequencies yield different reconstructions, and that the signal artificially contains very high frequencies as implied by the stair step of the sample boundary. These problems apply to any quantization of a continuous signal, such as images, where a scene is binned into pixels. Simple binning through sampling light intensity at the center of each pixel of a sensor yields speckling and pixelation because adjacent pixels differ too much, and the problem rapidly exacerbates when the image undergoes other processing, such as scaling. (Rosenfeld and Kak 1982). Anti-aliasing counters these problems through the use of any of a number of techniques. The analogy to spatial binning is obvious, where the contribution of a data point to a bin would not be all-or-nothing, but rather weighted depending upon distance from the center of the bin and including contributions from surrounding bins. This would prevent apparent patterns from changing as bin partitions change. There is extensive literature in sampling theory (e.g., Berry 1962; Berry and Baker 1968; Lounsbury, Sommers, and Fernald 1981; Anselin 1988; Maling 1988) that could inform improvements to spatial binning.

There is also ample room for research on the cognition of reading spatial bins. While we have pointed out possible pitfalls in apprehending spatial binning in a projected world, we have only assumed that mapmakers and map readers equate count and density measures. Perhaps this is our own cognitive bias, but considering previous research suggesting that map readers struggle to identify and incorporate projection distortion in interpreting maps (e.g., Anderson and Leinhardt 2002; Battersby 2009), we suspect map readers have trouble distinguishing the planar area (appearance of the bins) from the spherical areas they represent, which will lead to serious misinterpretation of spatial patterns. We also note that the purpose of binning is precisely to present constant shape and size in order to remove those variables in the mark so that the reader can focus on the measure the mark provides. Therefore when the size and shape do not mean what they apparently mean, the binning technique's *raison d'être* is defeated. Along this line of research, we need to understand how individuals interpret patterns within regular bins that do not represent constant spherical area, and with irregular bins that do represent constant spherical area, but are distorted in the planar map view.

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